

Enhanced winning in a competing population by random participationK. F. Yip,¹ T. S. Lo,¹ P. M. Hui,¹ and N. F. Johnson²¹*Department of Physics, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong*²*Physics Department, University of Oxford, Clarendon Laboratory, Oxford OX1 3PU, United Kingdom*

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We study a version of the minority game in which one agent is allowed to join the game in a random fashion. It is shown that in the crowded regime, i.e., for small values of the memory size m of the agents in the population, the agent performs significantly well if she decides to participate the game randomly with a probability q and she records the performance of her strategies only in the turns that she participates. The information, characterized by a quantity called the inefficiency, embedded in the agent's strategies performance turns out to be very different from that of the other agents. Detailed numerical studies reveal a relationship between the success rate of the agent and the inefficiency. The relationship can be understood analytically in terms of the dynamics in which the various possible histories are being visited as the game proceeds. For a finite fraction of randomly participating agents up to 60% of the population, it is found that the winning edge of these agents persists.

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I. INTRODUCTION

The self-organization of an evolving population consisting of agents competing for limited resource is an important problem in the science of complex systems and has potential applications in areas such as economics, biological, engineering, and social sciences [1,2]. The bar-attendance problem proposed by Arthur [3,4], for example, constitutes a typical setting of such system in which a population of agents decide whether to go to a bar with limited seating capacity for pleasure. The agents are informed of the attendance in past weeks, and hence the agents share common information, interact through their actions, and learn from past experience. The problem can be simplified by considering binary games, either in the form of the minority game (MG) [5] or in a binary-agent-resource game [6]. For low resource level in which there are more losers than winners, the minority game proposed by Challet and Zhang [5] represents a simple, yet nontrivial, model of competing populations.

The MG comprises an odd number N of agents. At each time step, the agents independently decide between option 0 and option 1. The winners are those who have chosen the minority side. The agents learn and adapt by evaluating the performance of their strategies, which map the global information, i.e., records of the most recent m winning options, to an action. A characteristic quantity in MG is the standard deviation σ of the number of agents making a particular choice. This quantity reflects the performance of the population as a whole in that a small σ implies on average more winners per turn, and hence a higher success rate per turn per agent. In MG, σ exhibits a nonmonotonic dependence on the memory size m of the agents [7–9]. When m is small, there is much overlap between the agents' strategies. This crowd effect [6,10,11] leads to a large σ , implying the number of losers is high. This is the crowded or efficient phase of MG. In the inefficient phase where m is large, σ is moderately small and the agents perform better than deciding randomly.

Two important questions in MG and other multi-agent based models are that (i) how one can possibly suppress σ

and lead to a better performance of population as a whole and (ii) how an individual agent can possibly outperform other agents without giving the agent too much extra capability. The former is clearly crucial from the point of view of a governing body in improving a society's performance, and the latter is important from the viewpoint of the agents. Much attention on the MG have been focused on the first question, and several models with a suppressed σ have been proposed. The thermal model of MG [12], for example, allows the agents to use strategies other than the best-scoring one with a probability taking on the form of a Boltzmann factor. It was found that σ can be suppressed significantly in the crowded phase, as the probabilistic usage of strategies reduces the crowd effect [13]. Alternatively, the use of a personal information [14] for decisions, e.g., based on each agent's record of her own actions, instead of a globally announced information provides another mechanism for suppressing the crowd effect. In the evolutionary minority game proposed by the present authors [15,16], the agents can adapt their behavior by adjusting their probability of following a strategy common to all agents. It was shown that the agents would self-organize in such a way that σ is small. Recently, Sysi-Aho *et al.* [17] proposed a genetic algorithm for the evolution of the agents and found that the fluctuations are reduced. A common feature of these models is that the way through which the agents adapt to past performance is significantly altered, as compared with the MG.

In the present work, we focus on the question of how individual agents may outperform their competitors. Our model is motivated by realistic behavior of ordinary people. Taking the situation of an agent buying or selling stocks, for example. He may not enter the market to sell or buy stocks every day. In addition, people tend to learn from their hands-on experience, i.e., they tend to learn in the turns that they actually participate. When they trade, they decide based on updated information. Based on these common observations, we propose a model in which an agent or a fraction of agents participate in the MG with a probability q per turn and these agents register the performance of their strategies *only*

in the turns that they participate. Strikingly, it is found that the winning probability of these randomly participating agents (henceforth referred to as RPA) are significantly enhanced, compared with the other agents who enter the MG every turn.

The plan of the paper is as follows. In Sec. II, we introduce our model. Results of extensive numerical studies for one randomly participating agent with $q=0.5$ in a population are presented in Sec. III. To explore the underlying physics of the enhanced success rate of the agent, we study the statistics of the bit strings observed by the agent. Results on the information observed, which is described by a quantity called the inefficiency ε , are presented in Sec. IV. It is found that a nontrivial relationship exists between the success rate of the agent in a run with ε of that run. A theoretical analysis on the observed relationship is also given in Sec. IV. In Sec. V, we present numerical results for $q \neq 0.5$ and generalize our discussion on the relationship between the success rate and ε to arbitrary values of q . Results for many RPAs in a population are presented in Sec. VI, together with results comparing the performance of RPAs and random agents who participate and decide randomly. It is found that the RPAs, with their scheme of assessing their strategies, consistently perform better than random agents. Section VII gives a summary of the results.

II. THE MODEL

The basic MG [5] comprises N agents competing to be in a minority group at each time step. The only information available to the agents is the history. The history is a bit string of length m recording the minority option for the most recent m time steps. There are a total of 2^m possible history bit strings. At the beginning of the game, each agent picks s strategies, with repetition allowed. They make their decisions based on their strategies. A strategy is a look up table with 2^m entries giving the predictions for all possible history bit strings. Since each entry can either be 0 or 1, the whole strategy pool contains 2^{2^m} strategies. Adaptation is built in by allowing the agents to accumulate a merit (virtual) point for each of her strategies as the game proceeds, with initial merit point set to zero for all strategies. After each turn, one (virtual) point is assigned to the strategies that would have predicted the winning minority option. At each turn, the agent follows the prediction of her best-scoring strategy. A random choice will be made for tied strategies.

In the present model, we consider a population of N agents in which there are a number N_{RPA} of randomly participating agents. These agents are allowed to join the game with a probability q , i.e., a RPA has a probability q of joining the game in each turn and a probability of $1-q$ of staying out of the game in a turn. The other $(N-N_{RPA})$ agents, as in the MG, participate every turn. For all agents, they decide in the same way as in the MG when they participate, i.e., they decide based on the best-scoring strategy that they hold at the moment of decision and follow the prediction of the strategy for the given most recent m winning options. In the case that the two possible options are chosen by the same number of agents, a case that may occur when RPAs are

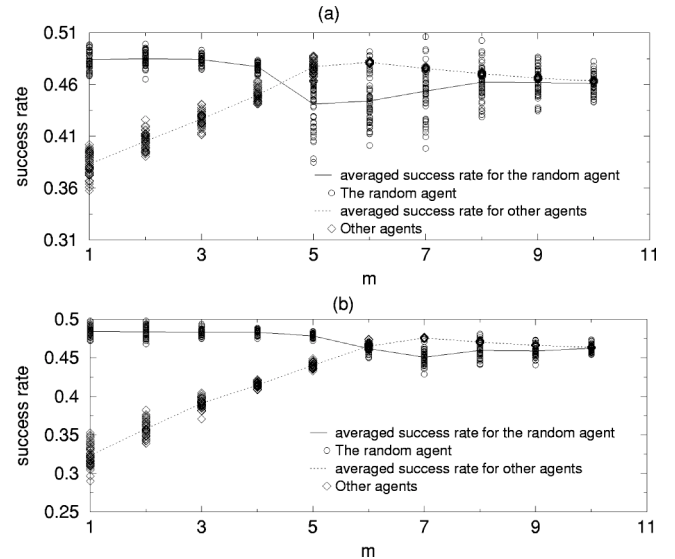


FIG. 1. The success rates R of the randomly participating agent and the other $(N-1)$ agents as a function of m for $N=101$, $q=0.5$ and for (a) $s=2$ and (b) $s=3$. For each value of m , the data correspond to 50 independent realizations. The lines represent an average over 50 runs. For $m < m_o$, an RPA performs significantly better than the other agents.

present, the winning outcome is decided randomly.

The RPAs differ from the other agents in the following way. Besides joining the game with probability q , they reward the virtual points to the strategies that they hold only in the turns that they participate. For the turns that a RPA decides not to play, no virtual points are awarded to her strategies, regardless of the outcome. These RPAs, therefore, carry the typical features of ordinary people as described in the preceding section. For $q=1$ or $N_{RPA}=0$, the present model reduces to the MG.

III. RESULTS FOR ONE RANDOMLY PARTICIPATING AGENT: $q=0.5$

We have performed extensive numerical simulations on our model. First, we study the case of one RPA in a population with $q=0.5$ in which the RPA is participating randomly, e.g., deciding by tossing an unbiased coin. We consider systems of $N=101$ and 301 agents, with each agent holding $s=2,3$ strategies. The quantity of interest is the success rate, which is the winning probability of the agents. For the RPA, the success rate R is the ratio of the number of winning turns to the number of turns she has actually participated.

Figure 1 shows the success rate of the RPA, together with the success rate of the other $(N-1)$ agents as a function of the memory size m for a system with $N=101$ agents for $s=2$ [Fig. 1(a)] and $s=3$ [Fig. 1(b)]. For each value of m , 50 independent runs with different initial random distributions of strategies among the agents are carried out. The most striking feature of the results is that in the crowded phase where m is small, i.e., $2 \times 2^m < Ns$, the RPA performs *significantly better* than the agents who participate every turn. The spread of results

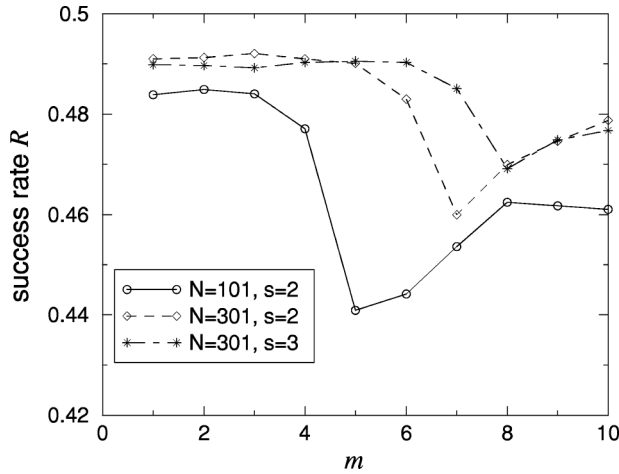


FIG. 2. The success rate R of the randomly participating agent as a function of m for three different systems: $q=0.5$, $N=101$ with $s=2$ and for $N=301$ with $s=2,3$. Each data point represents an average over 50 independent realizations.

in different runs is small and hence the enhanced success rate of the RPA is an intrinsic feature. For higher values of m in the inefficient phase of the MG, the success rate of the RPA is found to vary much from run to run, with an average success rate lower than that of the other agents. The crossover from one behavior to another occurs at a value m_o , which takes on a value close to the crossover from the crowded phase to the inefficient phase in the basic MG. For very high values of m , the success rates of the RPA and the rest of the population become identical. Similar behavior is also found for $s=3$. Comparing Figs. 1(a) and 1(b), the enhanced success rate of the RPA over the other agents in the range of small m is more pronounced for higher values of s , as the population does not perform collectively well in the basic MG as s increases for small m .

To test whether the dependence of success rate of the RPA on m is intrinsic, we carried out numerical simulation for a system with $N=301$ for $s=2,3$. The success rate of a RPA, averaged over 50 runs, is shown in Fig. 2, together with the results for $N=101$ and $s=2$ for comparison. In general, the average success rate depends only weakly on m for $m < m_o$. Note that the value of m_o depends on both N and s . Around m_o , the RPA's success rate drops and reaches a minimum before increasing with m again for $m > m_o$.

Qualitatively, the enhanced winning of a RPA comes from a successful escape in overadapting to the history created by action of the other agents. In the MG, adaptation is achieved by assessing the performance of each strategy as the game proceeds. In the crowded phase, it has been shown that the history bit string exhibits features with a periodicity of length 2×2^m [18]. For small values of m , agents in the population adapt too effectively to the history. When a particular history bit string occurs for the first time (or has occurred an even number of times in previous turns), the agents basically decide randomly. The outcome then leads to virtual points being awarded to those strategies which predicted the outcome for that particular history bit string. In the next occurrence of

the same history bit string, the virtual points awarded now lead to a crowd behavior [10] with a crowd of agents deciding on the same outcome as in the last occurrence. However, for small m , this crowd tends to be too big to win, due to the small strategy space and hence substantial overlap of strategies among agents. Hence, the outcome is opposite to that in the last occurrence of the same history bit string. Virtual points are then awarded. For the next occurrence of the same bit string, the situation is similar to that of the first occurrence. Since there are a total of 2^m possible history bit strings, it takes on average 2×2^m time steps to sample all the history bit strings twice, thus leading to the doubly periodic features. In graph-theoretical language, this is related to the path in the de Bruijn graph formed by the possible history bit strings. A crowd-anticrowd theory [10,11] can be formulated to explain the large standard deviation σ in the number of agents making a particular decision over time for small m in terms of the crowd effect. A large fluctuation implies fewer winners per turn and hence a low success rate. The success rates shown in Fig. 1 for the other agents are strongly correlated with σ . For the RPA, the virtual merit points of her strategies are different from that of the other agents, as she rewards the strategies only in the turns she participates. Therefore, the RPA does not fully adapt to the information created by the other agents, and hence does not become part of the crowd. This gives the RPA the ability to avoid over adaptation in the crowded phase and an winning edge over the other agents. We have checked that if the random agent keeps record of the performance of her strategies for the turns that she does not enter, the strategies still adapt to the global history and no enhanced success rate results, even she participates randomly.

IV. INEFFICIENCY AND SUCCESS RATE

A. Numerical results

To explore deeper into the underlying physics for the enhanced success rate of the RPA, we study the statistics of the bit strings in the series consisting of the outcomes for the turns that the RPA has participated. This series of bit strings corresponds to the one with which the strategies of the RPA are assessed for their performance. To quantify our discussion, we focus on the behavior in the range of small m and look at the probability of a winning outcome of 1 following a given bit string of m bits. We define the inefficiency ε [8,19,20] as follows:

$$\varepsilon = \frac{1}{2^m} \sum_{i=1}^{2^m} |P(1|i(m)) - 1/2|, \quad (1)$$

where the sum is over all 2^m possible m -bit strings and $P(1|i(m))$ is the conditional probability that a given m -bit string labeled by i is followed by an outcome 1 in a long series of the RPA bit strings in a particular realization of the model [21]. The inefficiency ε measures the information left in the history bit strings that the RPA uses to assess her strategies. Note that for the MG in the crowded phase, $\varepsilon=0$ [7], indicating that there is no information left in the history bit strings.

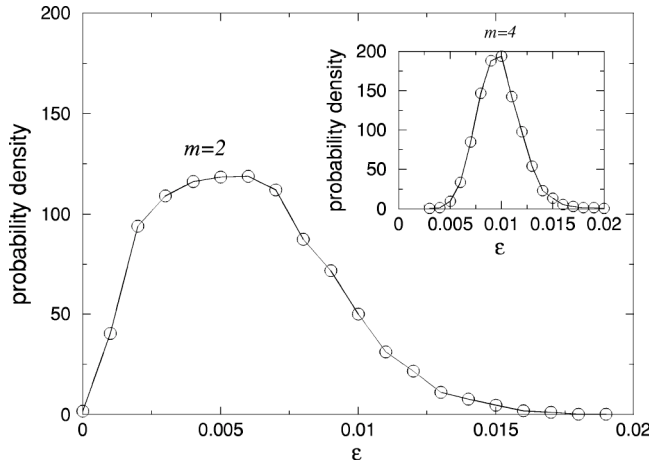


FIG. 3. The probability density of inefficiency ε for $N=101$, $m=2$, $s=2$, and $q=0.5$. The inset shows the probability distribution of ε for $m=4$. Each distribution is obtained from the results of 10 000 independent runs.

Each realization (run) of the model gives one value of ε . Figure 3 shows the distribution of the values of ε over 10 000 independent runs with different initial conditions for $m=2$ and $m=4$ (inset) and $q=0.5$. Strikingly, $\varepsilon \neq 0$ in general for the bit strings specific to the RPA. The distribution shows a peak at finite ε with the most probable value of ε increases with m . We have checked that the inefficiency of the bit string in a run used by the other agents takes on a value very close to zero.

From Figs. 1 and 3, there is a spread of success rate for different runs that is associated with a spread in the values of ε . For each run of our model, there is a success rate R and a value of ε . It is therefore interesting to explore the correlation, if any, between R and ε in a run. Figure 4 shows a plot of the success rate R against the inefficiency ε for a system with $N=101$, $s=2$, $m=2$, and a participating probability $q=0.5$ for the RPA. Each data point on the plot represents the success rate and the value of inefficiency in a run. There are 10 000 data points on the plot, corresponding to 10 000 independent runs. From the scattered data points, there emerges a straight line consisting of a fraction of ~ 0.06 of the total number of runs [22]. For these runs, the success rate R is related to the inefficiency ε by $R=0.5-\varepsilon$. The inset of Fig. 4 shows the results for $m=4$ in which the RPA holds two identical strategies, the implication of which will be discussed in the following section.

B. Theoretical analysis

The emergence of the relationship between the success rate and the inefficiency can be understood by invoking the doubly periodic feature in MG, as described in the last section. Let t_{even}^{μ} (t_{odd}^{μ}) be a set consisting of the turns in a history series that a particular history μ occurred an even (odd) number of times from the beginning of the game just before the moment of decision with history μ . Let $P(\tau|t_{even}^{\mu})$ [$P(\tau|t_{odd}^{\mu})$] be the probability that the outcome is τ ($\tau=0,1$)

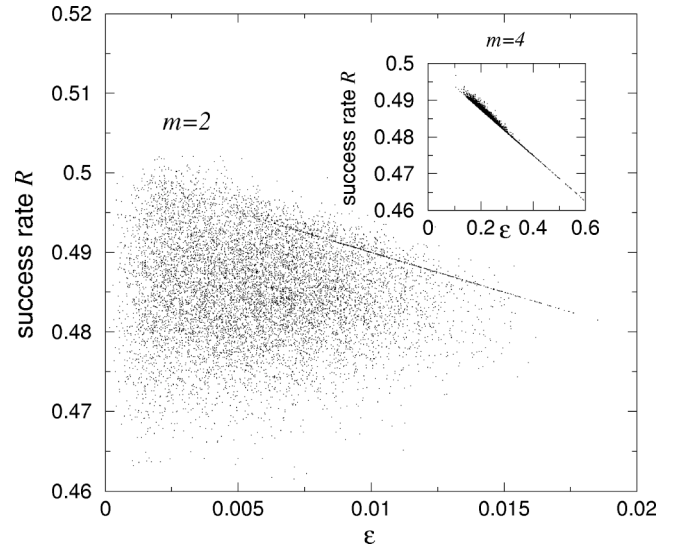


FIG. 4. The success rate R against ε in the bit string which the RPA uses to assess the performance of her strategy. The parameters are $N=101$, $s=2$, $m=2$, and $q=0.5$ for the RPA. There are 10 000 data points on the plot corresponding to 10 000 independent runs. Note that the data points lead to the emergence of a line. The inset shows the results for $m=4$ in which the RPA holds two identical strategies.

at a turn t that belongs to t_{even}^{μ} (t_{odd}^{μ}). Double-periodicity implies $P(\tau|t_{even}^{\mu}) = \frac{1}{2}$ and $P(\tau|t_{odd}^{\mu}) = 1 - \tau$. To proceed, we note that the straight line in main panel of Fig. 4 consists of about 600 data points out of a total of 10 000 runs. This amounts to a fraction of $1/(2^{2^m}) = 1/16$ for $m=2$, which is the fraction of agents holding two identical strategies in a population. Therefore, the runs on the straight line corresponds to the cases in which the RPA picks two identical strategies, i.e., effectively one strategy. This is further confirmed numerically in the inset of Fig. 4 in a game of $m=4$ in which the RPA is restricted to have two identical strategies. We further note that it is the RPA that generates the inefficiency ε . This is related to the fact that, unlike an outsider who observes but not participates, an agent's action affects the outcome, and hence her own success rate. This is the agent's "market impact." Consider, for example, the case that the other $(N-1)$ agents are equally split between the two options. The outcome will always be the opposite of that of the remaining agent. For a RPA, however, this market impact effect is important only for the turns $t \in t_{even}^{\mu}$. For $t \in t_{odd}^{\mu}$, the difference in the number of agents taking the two options is so large for a single RPA to affect the outcome.

Consider a RPA with two identical strategies. If the agent decides to enter the game in a turn $t \in t_{even}^{\mu}$ there is a small probability P in these turns that the other agents are evenly divided between option 0 and option 1. In this case, the outcome is determined by the strategy of the RPA. If her strategy predicts option τ , the outcome will be $(1 - \tau)$ and the agent will definitely lose. In the next occurrence of the history μ , the outcome must be τ due to the doubly periodic feature. As a result, the RPA will definitely win if she decides to participate. The success rate of the agent is then given by [23]

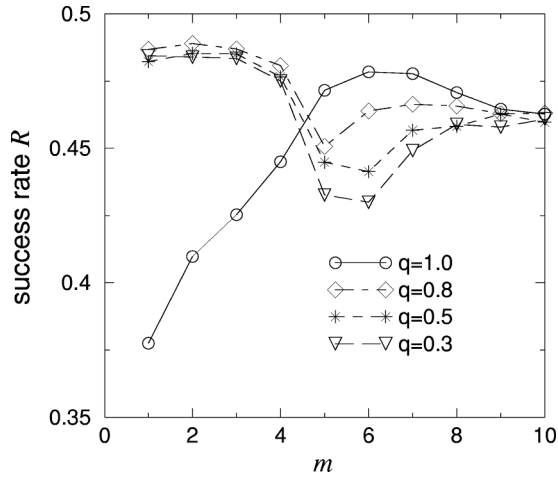


FIG. 5. The average success rate of the RPA as a function of m for a system with $N=101$, $s=2$ for different values of $q = 0.3, 0.5, 0.8, 1$. Each data point is an average over 50 independent runs. The lines are guide to eye.

$$R = (1 - P - qP) \times 1/2 + P \times 0 + qP \times 1 = \frac{1}{2} - \frac{1-q}{2}P, \quad (2)$$

where the last two terms in the first line correspond to the two cases discussed above, and the factor $(1 - P - qP)$ is the probability of occurrence of cases other than those included in the last two terms.

The inefficiency ε can be calculated using Eq. (1) in a similar manner. There is a probability P in the turns that the RPA participates that the RPA must lose for $t \in t_{\text{even}}^{\mu}$, with an outcome $(1 - \tau)$. If the agent participates in the next occurrence of that particular history μ , the outcome will be τ . Consequently, the probability for the RPA to observe an outcome τ is

$$P_{\tau} = (1 - P - qP) \times 1/2 + P \times 0 + qP \times 1 = \frac{1}{2} - \frac{1-q}{2}P, \quad (3)$$

where τ takes on either 0 or 1. It follows from Eq. (1) that

$$\varepsilon = \frac{1}{2^m} \sum_{\mu=1}^{2^m} |P(1|t^{\mu}) - 1/2| = \frac{1-q}{2}P. \quad (4)$$

Combining Eqs. (2) and (4), we have

$$R = \frac{1}{2} - \varepsilon, \quad (5)$$

as observed numerically for runs in which the RPA holds two identical strategies.

V. ONE RANDOMLY PARTICIPATING AGENT: $q \neq 0.5$

Figure 5 shows the results of the success rate, averaged over 50 runs for each data point, of a RPA as a function of m

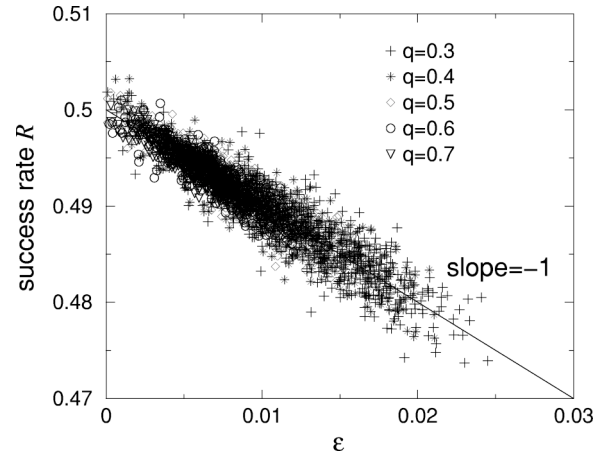


FIG. 6. The success rate R against ε with $N=101$, $s=2$, and $m=2$ for five different values of $q=0.3, 0.4, 0.5, 0.6, 0.7$. The RPA holds two identical strategies. For each value of q (labeled by a symbol), 500 independent runs are carried out. The solid line represents the relationship $R=0.5-\varepsilon$.

for a system with $N=101$, $s=2$ for different values of participation probability q ($q=0.3, 0.5, 0.8$). For $q=1$, the success rate of the RPA is the same as that of the other agents and the results are identical to that of the MG. It is noted that for small values of m , i.e., in the crowded phase, the enhanced success rates for $q \neq 1$ take on similar values, all of which are significantly higher than that of the other agents (reasonably represented by the $q=1$ results). For values of m in the inefficient phase of the MG, a higher participation probability gives a higher success rate. However the success rate is still lower than that of the other agents.

The argument in the preceding section can be readily generalized to arbitrary values of $q \neq 1$. In this case, the success rate of a RPA holding repeated strategies is given by

$$R(q) = \frac{1}{2} - \varepsilon(q), \quad (6)$$

i.e., taking on the same form as Eq. (5). The inefficiency $\varepsilon(q)$, similar to Eq. (4), is given by

$$\varepsilon(q) = \frac{1-q}{2}P(q), \quad (7)$$

where $P(q) = T_c(q)/T$ with $T_c(q)$ being the number of turns in a run of T turns ($T \gg 1$) that the other $(N-1)$ agents are evenly split between the two decisions and the RPA participates. The result suggests that for different values of q , a plot of the success rate R against $\varepsilon(q)$ will have data points clustered on a straight line with slope -1 . This is indeed the case as shown for data obtained with five different values of q in Fig. 6. For each value of q , 500 independent runs are carried out, and there are 2500 data points on the plot. The data clearly exhibit the behavior suggested by Eq. (6).

It is interesting to explore the distribution of inefficiency for values of q away from $q=1$. Figure 7 shows the probability density of inefficiency for four different values of q

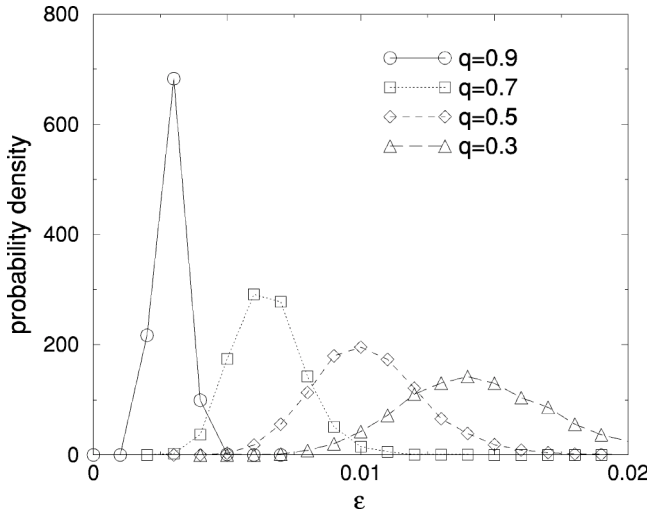


FIG. 7. The probability density of inefficiency ε for the RPA in a system with $N=101$, $s=2$, $m=2$ for four different values of $q = 0.3, 0.5, 0.7, 0.9$. Other parameters are $N=101$, $m=4$, and $s=2$. For each value of q , the distribution is obtained from 1000 independent runs.

($q=0.3, 0.5, 0.7, 0.9$) in a system with $N=101$, $m=4$, and $s=2$. For each value of q , the distribution is obtained from 1000 runs. It should be pointed out that the distribution for the basic MG ($q=1$) is sharply peaked near $\varepsilon=0$. As q takes on values gradually away from unity, the distribution of inefficiency gradually spreads wider with the most probable value of the inefficiency increases as the deviation of q from unity increases. Therefore, the enhanced success rate is accompanied by a nonvanishing inefficiency in the range of small m .

VI. MANY RANDOMLY PARTICIPATING AGENTS

It is important to investigate whether the enhanced success rate of a RPA persists when more RPAs are present in a population. Figure 8 shows the averaged success rate as a function of the fraction, N_{RPA}/N , of RPAs in a population with $N=101$, $s=2$, $m=2$, and $q=0.5$. The dashed line gives the success rate of the RPAs and the solid line gives the success rate of the other agents, for given N_{RPA}/N . Most strikingly is that for a wide range of N_{RPA}/N up to about 60%, the success rates of the RPAs and the rest of the population are basically insensitive to N_{RPA} . Over this range of N_{RPA}/N , the success rate of the RPAs are much higher than that of the other agents, as in the case of $N_{RPA}=1$. The results show that the system is still dominated by the crowd effect for $N_{RPA}/N < 0.6$, and the RPAs achieve a higher success rate by avoiding themselves from the crowd. For $N_{RPA}/N \geq 0.6$, the success rate of the RPAs decreases with N_{RPA}/N and that of the other agents increases. It is expected that the fraction of RPA over which the RPAs outperform the other agents may depend on the value of q . For $q=0.5$ discussed here, half of the RPAs participate in each turn on the average. Thus the number of participating agents effectively reduces. Up to a fraction of 60%, the majority of the participating

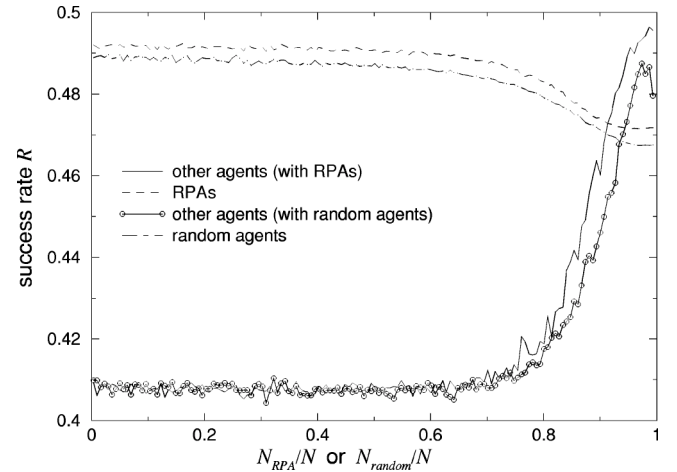


FIG. 8. The average success rate as a function of the ratio N_{RPA}/N of RPAs in the population with $N=101$, $s=2$, $m=2$, and $q=0.5$. A pair of lines show the success rate of the RPAs (dashed line) and the other agents (solid lines). For comparison, another pair of lines show the success rate of random agents (dot-dashed line) and the other agents (solid line with symbols) as a function of the ratio N_{random}/N . Each data point represents an average over 50 independent realizations.

agents in a turn are the other agents. Thus, the system is still influenced by the actions of the other agents.

VII. CONCLUSION

We considered the success rate of a number of randomly participating agents in a population competing for limited resource. The RPAs participate with a probability q and they assess the performance of their strategies only in the turns that they participate. Extensive numerical calculations were carried out. For one RPA in a population, it was found the RPA has a higher success rate than the other agents in the crowded phase. Rewarding the strategies only in the turns of participation avoids the RPA from overadaptation to the outcomes produced by the actions of the other agents. The hidden information in the bit strings that a RPA used to reward her strategies was analyzed in terms of the inefficiency ε . In basic MG, $\varepsilon=0$ for all agents. In our model, the RPA sees $\varepsilon > 0$. Numerical data reveal a relationship between the success rate R and ε of the form $R=0.5-\varepsilon$, if the RPA holds two identical strategies. The relationship was explained in terms of the dynamics through which the possible history bit strings were visited. Results for $q \neq 0.5$ were also presented and discussed. The RPA has an enhanced success rate for $q \neq 1$, together with nonvanishing ε . For many RPAs in a population, it was found that the winning edge of the RPAs persists even for a population up to about 60% of RPAs for $q=0.5$.

It is also interesting to explore if it is random participation alone that leads to the enhanced success rate. We, therefore, consider the case of a fraction of agents N_{random}/N who participate with probability q and decide the action randomly, i.e., they do not use the most recent m outcomes for decisions. Results are shown in Fig. 8 for comparison. The suc-

cess rate of these random agents (dot-dashed line), together with that of the other agents (solid line with symbols) are shown as a function of N_{random}/N . The results show that the success rate of the RPAs is *consistently higher* than that of the random agents for all ratios in the population. We have also checked that the RPAs perform better than random agents for all values of m . Thus, while random participating does lead to a better performance by avoiding the crowd, the scheme of rewarding virtual points to the RPAs and making

use of the virtual points in decision making gives an additional advantage for a further improved performance.

ACKNOWLEDGMENTS

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